



The effect of irreversibilities on solar Stirling engine cycle performance

M. Costea^{a,*}, S. Petrescu^a, C. Harman^{b, 1}

^a*Department of Applied Thermodynamics, Polytechnic University of Bucharest, Splaiul Independentei 313, Bucharest, Romania*

^b*Department of Mechanical Engineering, Duke University, Durham, NC 27706, USA*

Abstract

The purpose of this study is to determine the effect of pressure losses and actual heat transfer on the performance of a solar Stirling engine. The model presented includes the effects of both internal and external irreversibilities of the cycle. The solar Stirling engine is analyzed using a mathematical model based on the first law of thermodynamics for processes with finite speed, with particular attention to the energy balance at the receiver. Pressure losses, due to fluid friction internal to the engine and mechanical friction between the moving parts, are estimated through extensive and rigorous use of the available experimental data. The results of this study show that the real cycle efficiency is approximately half the ideal cycle efficiency when the engine is operated at the optimum temperature. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Solar stirling energy; Irreversible cycle; Pressure losses; Actual heat transfer; Fluid friction; Mechanical friction

1. Introduction

Evaluation of the losses due to the irreversibilities in the Stirling cycle is a topic of significant interest to those concerned with the analysis and performance of thermal machines.

* Corresponding author. Tel.: +40-1-410.45.85/339; fax: +40-1-410.42.51.

E-mail addresses: mcos@theta.thermo.pub.ro (M. Costea), cmh@egr.duke.edu (C. Harman)

¹ Tel.: +919-660-5310; fax: +919-660-8963

Nomenclature

A	absorbtivity
a	coefficient
C	concentration ratio
c_v	constant volume specific heat
D	diameter
E	insolation
m	mass of the gas
N	number of gauzes of the matrix, number of regenerators per cylinder
n_r	rotation speed
p	pressure
Δp	pressure loss
$\dot{Q}$	heat transfer rate
R	gas constant
S	surface area
s	stroke
T	temperature
ΔT	temperature difference
u	convective heat transfer coefficient
V	volume
$\dot{W}$	power output
w	piston speed
X	fraction of heat lost due to incomplete regeneration

Greek symbols

ε	compression ratio, emisivity
γ	specific heat ratio
η	efficiency
η'	efficiency defined in Eq. (6)
ρ	density of the gas
σ	constant of Stefan–Boltzmann

Subscripts

aver	average
C	cylinder, related to the Carnot cycle
c	related to the working fluid temp.
cav	cavity
conc	concentration
cpr	compression
exp	expansion
ext	external
f	friction

g	gas
H	source
L	sink
M	material
mir	mirror
o	ambient
open	receiver opening
R	regenerator
Rec	receiver
SE	Stirling engine
v	volumetric
w	related to the piston speed or to the working fluid temperature
1	state in cycle
II	related to the Second Law

It is the focus of this study. Our studies and recent investigations by others [1,2] have found that irreversibilities in the thermodynamic cycle have significant importance in predicting the performance of Stirling engines. Efforts have been made over the past several years to better understand these irreversibilities related losses [3,4]. This has resulted in a number of models for the analysis and optimization of Stirling machines that include the effect of irreversibilities in the cycle. However, analyses that are based on the conventional entropy or exergy techniques do not relate the irreversibilities to the physical phenomena that cause them. The model presented here directly connects the irreversibilities to the operation of the cycle at finite speed. It provides a clearer understanding of the loss mechanisms and relates them more quantitatively to the thermodynamic irreversibility terms. This in turn provides increased insight to the loss mechanism.

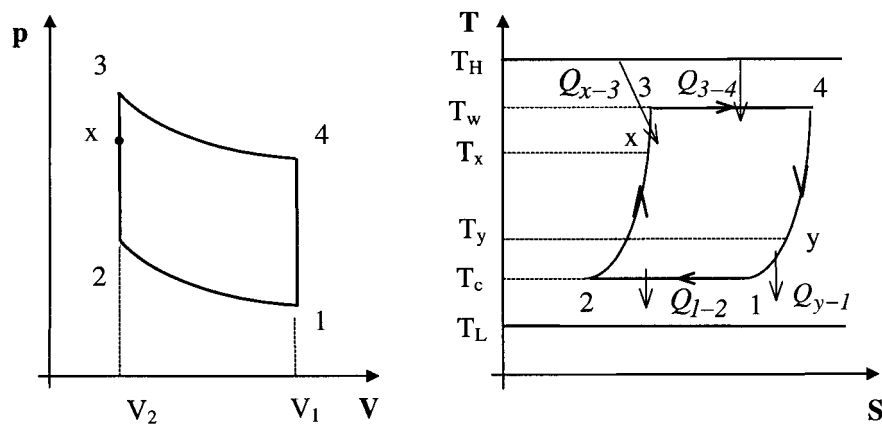


Fig. 1. Stirling engine cycle with irreversibilities.

The model presented is an extension of previous work [5,6] and includes the effects of both internal and external irreversibilities. Attention in the analysis presented is given to the effects of (1) heat transfer through a temperature difference at the source and sink, (2) incomplete heat regeneration, (3) piston speed and (4) effects of internal mechanical and fluid friction. This presentation extends previous work by basing the analytical pressure loss prediction due to internal fluid and mechanical friction on actual operating data and by basing the heat transfer rate to the engine on the available heat at the solar source.

2. Analysis of the solar Stirling engine cycle with irreversibilities

2.1. The Stirling engine cycle with irreversibilities

The ideal Stirling engine cycle is shown on p - V and T - S diagrams in Fig. 1. The T - S diagram is modified to include the effects of heat transfer through a temperature difference at the source and sink and for incomplete regeneration.

Additional heat from the external source is shown to be needed in the process, Q_{x-3} , due to incomplete regeneration. Similarly, the unregenerated heat is shown being rejected in the process, Q_{y-1} . Fluid friction of the gas as it passes through the regenerator causes most of the pressure loss.

2.2. Mathematical model

The solar Stirling engine is analyzed using a mathematical model based on the first law of thermodynamics for processes with finite speed, with particular attention to the energy balance at the receiver, using what we call “the direct method” [7–10].

The net power output of the engine is:

$$\dot{W} = \eta_{SE} \cdot \dot{Q}_H = \left(1 - \frac{T_L + \Delta T_L}{T_H - \Delta T_H} \right) \cdot \eta_{II,irrev} \cdot \dot{Q}_H \quad (1)$$

with $\Delta T_H = T_H - T_w$, and $\Delta T_L = T_c - T_L$.

In Eq. (1) the engine efficiency is separated in two terms, the second-law efficiency, $\eta_{II,irrev}$ [11] and the Carnot cycle efficiency:

$$\eta_{C,\Delta T} = 1 - \frac{T_L + \Delta T_L}{T_H - \Delta T_H} \quad (2)$$

The heat flux transferred to the working gas in the receiver of the engine has the form:

$$\begin{aligned} \dot{Q}_H &= [mR(T_H - \Delta T_H) \cdot \ln \varepsilon_v + X \cdot mc_v(T_H - \Delta T_H - T_L - \Delta T_L)] \cdot n_r \\ &= \frac{p_1 V_1}{T_L + \Delta T_L} (T_H - \Delta T_H) \left[\ln \varepsilon_v + \frac{X}{\gamma - 1} \cdot \eta_{C,\Delta T} \right] \cdot n_r \end{aligned} \quad (3)$$

The effect of each of the engine irreversibilities is evaluated independently [10] so the effect

of each is clear in the overall optimization procedure. Thus, the second-law efficiency has the form:

$$\eta_{\text{II,irrev}} = \eta_{\text{II,irrev}_{\text{int}(X)}} \cdot \eta_{\text{II,irrev}_{\text{int}(\Delta p)}} \quad (4)$$

where the effect of incomplete regeneration, internal mechanical and fluid friction and piston speed are given by:

$$\eta_{\text{II,irrev}_{\text{int}(X)}} = \frac{1}{1 + \frac{X}{(\gamma - 1) \cdot \ln \varepsilon_v} \cdot \eta_{C,\Delta T}} \quad (5)$$

$$\eta_{\text{II,irrev}_{\text{int}(\Delta p)}} = 1 - \frac{3\mu \cdot \sum(\Delta p_i)/p_1}{\eta' \cdot \frac{T_H - \Delta T_H}{T_L + \Delta T_L} \cdot \ln \varepsilon_v} \quad (6)$$

with

$$\eta' = \eta_{C,\Delta T} \cdot \eta_{\text{II,irrev}_{\text{int}(X)}} \quad (7)$$

$$\mu = 1 - \frac{1}{3 \cdot \varepsilon_v} \quad (8)$$

Pressure losses considered in the model are [7–10]:

$$\sum(\Delta p)_i = \Delta p_{\text{aver}_g} + \Delta p_f + \Delta p_w \quad (9)$$

Pressure losses, due to fluid friction internal to the engine were estimated through extensive and rigorous use of the available experimental data [12]:

$$\Delta p_{\text{aver}_g} = \Delta p_{\text{throt}_R} = \frac{15}{\gamma} \left(\frac{1}{2} \rho_R w_R^2 \right) N \quad (10)$$

where the speed of the gas in the regenerator is considered to be proportional to the average piston speed:

$$w_R = \frac{w}{2} \cdot \frac{D_C^2}{N_R D_R^2} \quad (11)$$

and the gas density in the regenerator, ρ_R , were calculated using the average values of temperature and pressure of the gas in the regenerator.

We note that the pressure losses in the heater and cooler were neglected. Thus, the term of Eq. (10) takes account only of the pressure losses due to the throttling of the gas flowing in the regenerator. This expression was found by an analytical analysis of the experimental data from [12, Fig. 5.7].

The mean effective pressure losses resulting from mechanical friction of the engine

components were estimated. The estimates are based on experimental data from internal combustion engine [13] adapted to actual Stirling machine operation:

$$\Delta p_f = \frac{(0.94 + 0.045 \cdot w) \cdot 10^5}{3\mu} \left(1 - \frac{1}{\varepsilon_v}\right) [\text{N/m}^2] \quad (12)$$

with w [m/s].

Pressure losses, due to the piston speed [7–10], actually to the pressure waves generated by the finite speed movement of the pistons, waves traveling with the speed of sound in the gas, are:

$$\Delta p_w = \frac{1}{2} \left(p_{\text{aver}_{\text{cpr}}} \cdot \frac{a \cdot w_{\text{cpr}}}{c_{\text{cpr}}} + p_{\text{aver}_{\text{exp}}} \cdot \frac{a \cdot w_{\text{exp}}}{c_{\text{exp}}} \right) \quad (13)$$

where

$$c = \sqrt{3RT} - \text{average molecular speed}; \quad (14)$$

$a = \sqrt{3\gamma}$ —coefficient which depends on the specific heat ratio; $w_{\text{cpr}} \cong w_{\text{exp}} = w$; $p_{\text{aver}_{\text{cpr}}}$, $p_{\text{aver}_{\text{exp}}}$ —average pressure during the compression and expansion of the gas respectively.

By combining Eqs. (9)–(14), the pressure losses becomes function of $\mathbf{w}$ and $\mathbf{T}_H$:

$$\begin{aligned} \sum(\Delta p_i) = & \frac{15}{\gamma} \left[\frac{1}{2} \frac{p_1 \cdot (1 + \varepsilon_v)}{R \cdot (T_L + \Delta T_L)} \cdot \frac{w^2}{4} \right] \cdot N \cdot \left(\frac{D_C^2}{N_R D_R^2} \right)^2 + \frac{(0.94 + 0.045 \cdot w) \cdot 10^5}{3\mu} \\ & \cdot \left(1 - \frac{1}{\varepsilon_v} \right) + \frac{1}{2} w \cdot p_1 \cdot \frac{\varepsilon_v}{\varepsilon_v - 1} \\ & \cdot \ln \varepsilon_v \cdot \sqrt{\frac{\gamma}{R}} \cdot \frac{1}{\sqrt{T_L + \Delta T_L}} \cdot \left[1 + \sqrt{\frac{T_H - \Delta T_H}{T_L + \Delta T_L}} \right] \end{aligned} \quad (15)$$

Taking account of the relation between the piston speed and the rotation speed (s-stroke):

$$n_r = \frac{w}{2s} \quad (16)$$

Eq. (15) becomes a function of $\mathbf{n}_r$ and $\mathbf{T}_H$.

When Eqs. (3)–(8) and (15) are substituted into Eq. (1), the net power output can be shown to be a function only of $\mathbf{n}_r$ and $\mathbf{T}_H$.

The second assumption of the model is that the heat flux received by the engine is equal to the heat flux supplied by the solar receiver:

$$\dot{Q}_{\text{avail}} = \dot{Q}_H \quad (17)$$

where $\dot{Q}_H$ is given by Eq. (3) as a function of the same variables $\mathbf{n}_r$ and $\mathbf{T}_H$.

The term $\dot{Q}_{\text{avail}}$ was determined for a dish mirror with cavity type receiver:

$$\dot{Q}_{\text{avail}} = \eta_{\text{Rec}} \cdot A_{\text{cav}} \cdot E \cdot C \cdot \eta_{\text{conc}} \quad (18)$$

where the receiver efficiency, η_{Rec} , depends on T_{H} and optical characteristics of the concentration system [5,6,14].

Finally, Eq. (2) imposes the rotation speed as a constraint in correlating the heat flux to the cycle gas with the heat supplied by the receiver. Upon substitution into Eq. (18):

$$\begin{aligned} & \frac{p_1 V_1}{T_L + \Delta T_L} \cdot (T_{\text{H}} + \Delta T_{\text{H}}) \cdot \ln \varepsilon_v \cdot \left[1 + \frac{X}{(\gamma - 1) \cdot \ln \varepsilon_v} \eta_{C, \Delta T} \right] \cdot n_r \\ &= \frac{\pi D_{\text{mirr}}^2}{4} \cdot E \cdot \eta_{\text{conc}} \cdot A_{\text{cav}} \cdot \left[1 - \frac{\varepsilon_m \sigma T_{\text{H}}^4}{A_{\text{cav}} \cdot \eta_{\text{conc}} \cdot E \cdot C} - \frac{u_{\text{cav}}(T_{\text{H}} - T_0) S_{\text{cav}}}{A_{\text{cav}} \cdot \eta_{\text{conc}} \cdot E \cdot C \cdot S_{\text{open}}} \right. \\ & \quad \left. - \frac{u_{\text{ext}}(T_{\text{ext}} - T_0) S_{\text{ext}}}{A_{\text{cav}} \cdot \eta_{\text{conc}} \cdot E \cdot C \cdot S_{\text{open}}} \right] \end{aligned} \quad (19)$$

where the results have been determined in previous work [5,6,14].

The analysis to accomplish the optimization of the solar Stirling engine results in a system of nonlinear equations that are solved using the method of Lagrangian undetermined multipliers.

3. Results

The preceeding analysis of the Stirling engine provides an expression for the optimum speed of engine rotation and for the power output, including the effect of irreversibilities, as a function of the source temperature. Calculations have been made for illustration using helium as the working fluid and with the following fixed parameters: $D_{\text{mirr}} = 6.5$ m, $d_{\text{Rec}} = 0.4$ m,

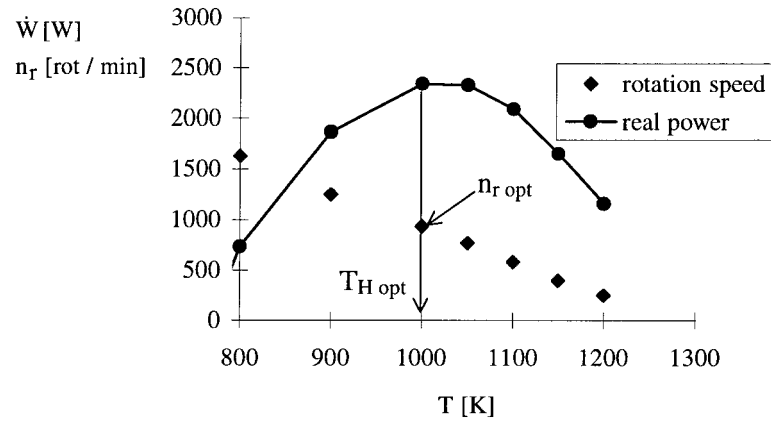


Fig. 2. Real power output and the corresponding rotation speed of the engine versus temperature (helium, $\Delta T = 25$ K, $d_{\text{Rec}} = 0.4$ m, $E = 750$ W/m²).

$\eta_{\text{conc}}=0.665$, $u_1=7 \text{ W/m}^2\text{K}$, $u_2=6 \text{ W/m}^2\text{K}$, $\varepsilon_m=0.8$ and $A_{\text{cav}}=0.94$, $E=750 \text{ W/m}^2$, $T_0=293 \text{ K}$, $\sigma=5.67 \text{ W/m}^2\text{K}^4$, $\varepsilon_v=5$, $X=0.2$, $s=0.1 \text{ m}$.

The results of the calculations are shown plotted in Fig. 2. The maximum value of the power output $\dot{W}[W]$ of the Stirling cycle engine indicates the optimum temperature of the source (in this example i_n , [rot/min] is seen to be at about 1000 K).

The efficiency of the Stirling engine operating with the same fixed parameters, both ideal and including the irreversibilities considered in this analysis, is shown in Fig. 3. This figure indicates that the ratio of actual to ideal efficiency increases as the temperature increases and that the ratio is approximately one half at the optimum temperature for maximum power output.

The effect of irreversibilities on each of the cited efficiencies as a function of source temperature is shown in Fig. 4. The fixed parameters are the same as in the previous illustrations.

It can be seen that for low temperature of the receiver the effect of pressure losses is important. The efficiency associated to the incomplete regeneration, $\eta_{\text{II, irrev, int}(X)}$, does not influence substantially the reversible cycle efficiency in comparison with the pressure losses in the range of the optimum temperature we have obtained ($T_H=950 \div 1100 \text{ K}$, see Fig. 2). This conclusion results from the calculation we have made for $X=0.2; 0.4; 0.6$.

The performance predicted (30% efficiency) in the analysis presented is in close agreement with actual performance data obtained experimentally from a Stirling engine powered by a solar dish [15].

4. Conclusion

An analysis of the solar Stirling engine with irreversibilities is presented that makes possible the prediction of the performance of actual solar Stirling engines operating under the same conditions.

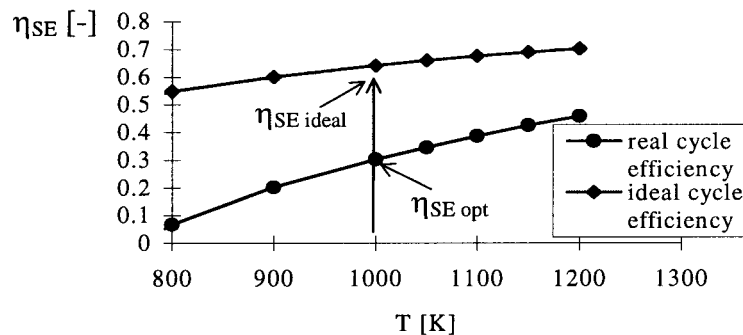


Fig. 3. Efficiency of the engine operating upon an ideal or an irreversible cycle versus temperature (helium, $\Delta T=25 \text{ K}$, $d_{\text{Rec}}=0.4 \text{ m}$, $E=750 \text{ W/m}^2$).

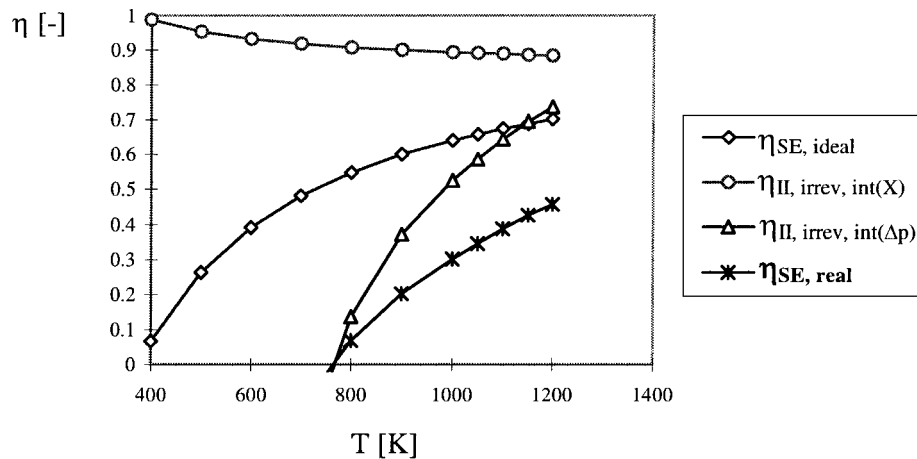


Fig. 4. Efficiencies associated with each class of irreversibility.

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